**P8, #10:** Formulate a dynamical system that models change exactly for the described situation.

You owe $500 on a credit card that charges 1.5% interest each month. You pay $50 each month and you make no new charges.

|  |  |
| --- | --- |
| bn | week |
| 500 | 0 |
| 457.5 | 1 |
| 414.3625 | 2 |
| 370.5779375 | 3 |
| 326.1366066 | 4 |
| 281.0286557 | 5 |
| 235.2440855 | 6 |
| 188.7727468 | 7 |
| 141.604338 | 8 |
| 93.72840305 | 9 |
| 45.1343291 | 10 |
| -4.188655968 | 11 |

**P17, #9:** The data in the accompanying table show the speed *n* (in increments of 5mph) of an automobile and the associated distance a*n* in feet required to stop it once the breaks are applied. For instance, *n=*6 (representing 6x5 = 30 mph) requires a stopping distance of a6 = 47 ft.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| an | 3 | 6 | 11 | 21 | 32 | 47 | 65 | 87 | 112 | 140 | 171 | 204 | 241 | 282 | 325 | 376 |

1. Calculate and plot the change a*n* versus *n.* Does the graph reasonably approximate a linear relationship?

The change in stopping distance vs n speed appears to be linear.

|  |  |  |  |
| --- | --- | --- | --- |
| n | speed | an | a(n+1)-an |
| 1 | 5 | 3 | 3 |
| 2 | 10 | 6 | 5 |
| 3 | 15 | 11 | 10 |
| 4 | 20 | 21 | 11 |
| 5 | 25 | 32 | 15 |
| 6 | 30 | 47 | 18 |
| 7 | 35 | 65 | 22 |
| 8 | 40 | 87 | 25 |
| 9 | 45 | 112 | 28 |
| 10 | 50 | 140 | 31 |
| 11 | 55 | 171 | 33 |
| 12 | 60 | 204 | 37 |
| 13 | 65 | 241 | 41 |
| 14 | 70 | 282 | 43 |
| 15 | 75 | 325 | 51 |
| 16 | 80 | 376 |  |

1. Based on your conclusions in part (a), find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against *n.* Discuss the appropriateness of the model.

Slope is roughly rise over run, so:

Rise of change = 51-3 = 48

Run (n) = 16-1 = 15

48/15 = 3.2

But not entirely sure why an vs n is important. Should it not be: change an vs an? Or:

This appears to be a curve, not a straight line…but assuming it is linear as per part a:

Difference equation in terms of an:

Rise (change in stopping distance) = 51-3 = 48

Run (stopping distance) = 325-3 = 322

48/322 = .15

So here is further proof that the line is indeed a curve and that a straight line would not be an optimal model.

**Page 34: #13**

Consider the spreading of a rumor through a company of 1000 employees, all working in the same building. We assume that the spreading of a rumor is similar to the spreading of a contagious disease (see example 3, section 1.2) in that the number of people hearing the rumor each day is proportional to the product of the number who have heard the rumor previously and the number who have no heard the rumor. This is given by

(1000-n)

where k is a parameter that depends on how fast the rumor spreads and n is the number of days. Assume k = .001 and further assume that four people initially have heard the rumor. How soon will all 1000 employees have heard the rumor?

If we’re rounding up, about 11 days. If not, 13 days.

|  |  |
| --- | --- |
| Day | People who've heard rumor |
| 0 | 4 |
| 1 | 7.984 |
| 2 | 15.90425574 |
| 3 | 31.55556614 |
| 4 | 62.11537852 |
| 5 | 120.3724368 |
| 6 | 226.25535 |
| 7 | 401.3192167 |
| 8 | 641.5813197 |
| 9 | 871.5360496 |
| 10 | 983.4970134 |
| 11 | 999.7276514 |
| 12 | 999.9999258 |
| 13 | 1000 |

**Page 55: #6**

An economist is interested in the variation of the price of a single product. It is observed that a high price for the product in the market attracts more suppliers. However, increasing the quantity of the product supplied tends to drive the price down. Over time, there is an interaction between price and supply. The economist has proposed the following model, where Pn represents the price of the product at year *n*, and Q*n* represents the quantity. Find the equilibrium values for this system.

If P = Pn+1 = Pn and Q = Qn+1 = Qn..

or…

checking…

100 = 100 - .1(500-500)

500 = 500 + .2(100-100)

So works out, although seems shortcutish

**Does the model make sense intuitively? What is the significance of the constants 100 and 500? Explain the significance of the signs of the constants -.01 and +.2.**

Yes, makes sense intuitively. I believe the 100 and 500 exist to translate/convert between the two categories of price (which would be in currency) and quantity (which is presumably in some metric or imperial scale). The -.01 allows price to decrease as quantity increases, and the +.2 allows quantity to increase as price also increases, both are the observed market forces described.

**Test the initial conditions in the following table and predict the long term behavior.**

|  |  |  |
| --- | --- | --- |
|  | **Price** | **Quantity** |
| **Case A** | **100** | **500** |
| **Case B** | **200** | **500** |
| **Case C** | **100** | **600** |
| **Case D** | **100** | **400** |

**Case A: 100/500,** these are the equilibrium values calculated earlier, and they appear to stay constant…

|  |  |  |
| --- | --- | --- |
| N | Price | Quantity |
| 0 | 100 | 500 |
| 1 | 100 | 500 |
| 2 | 100 | 500 |
| 3 | 100 | 500 |
| 4 | 100 | 500 |
| 5 | 100 | 500 |
| 6 | 100 | 500 |
| 7 | 100 | 500 |
| 8 | 100 | 500 |
| 9 | 100 | 500 |
| 10 | 100 | 500 |

**Case B: 200/500,** the market appears to be initially overloaded with quantity in this scenario, maxing out at about 658 units and driving the price to 0. Yet the market rebounds with price eventually recovering and driving quantity back down ad infinitum.

|  |  |  |
| --- | --- | --- |
| N | Price | Quantity |
| 0 | 200 | 500 |
| 1 | 200 | 520 |
| 2 | 198 | 540 |
| 3 | 194 | 559.6 |
| 4 | 188.04 | 578.4 |
| 5 | 180.2 | 596.008 |
| 6 | 170.5992 | 612.048 |
| 7 | 159.3944 | 626.16784 |
| 8 | 146.777616 | 638.04672 |
| 9 | 132.972944 | 647.4022432 |
| 10 | 118.2327197 | 653.996832 |
| 11 | 102.8330365 | 657.6433759 |
| 12 | 87.06869889 | 658.2099832 |
| 13 | 71.24770056 | 655.623723 |
| 14 | 55.68532826 | 649.8732631 |
| 15 | 40.69800195 | 641.0103288 |
| 16 | 26.59696907 | 629.1499292 |
| 17 | 13.68197616 | 614.469323 |
| 18 | 2.235043858 | 597.2057182 |
| 19 | -7.485527963 | 577.652727 |

**Case C: 100/600,** quantity first escalates with a corresponding drop in price. Then price recovers with corresponding drop in quantity, ad finitium.

|  |  |  |
| --- | --- | --- |
| N | Price | Quantity |
| 0 | 100 | 600 |
| 1 | 90 | 600 |
| 2 | 80 | 598 |
| 3 | 70.2 | 594 |
| 4 | 60.8 | 588.04 |
| 5 | 51.996 | 580.2 |
| 6 | 43.976 | 570.5992 |
| 7 | 36.91608 | 559.3944 |
| 8 | 30.97664 | 546.777616 |
| 9 | 26.2988784 | 532.972944 |
| 10 | 23.001584 | 518.2327197 |
| 11 | 21.17831203 | 502.8330365 |
| 12 | 20.89500838 | 487.0686989 |
| 13 | 22.1881385 | 471.2477006 |
| 14 | 25.06336844 | 455.6853283 |
| 15 | 29.49483561 | 440.698002 |
| 16 | 35.42503542 | 426.5969691 |
| 17 | 42.76533851 | 413.6819762 |
| 18 | 51.39714089 | 402.2350439 |
| 19 | 61.17363651 | 392.514472 |
| 20 | 71.92218931 | 384.7491993 |

**Case D: 100/400,** both quantity and price rise in parallel, with price reaching its peak first and decreasing while quantity continues to rise, then quantity decrease as the price slope starts its recovery. The quantity slope looks like the derivative of price.

|  |  |  |
| --- | --- | --- |
| N | Price | Quantity |
| 0 | 100 | 400 |
| 1 | 110 | 400 |
| 2 | 120 | 402 |
| 3 | 129.8 | 406 |
| 4 | 139.2 | 411.96 |
| 5 | 148.004 | 419.8 |
| 6 | 156.024 | 429.4008 |
| 7 | 163.08392 | 440.6056 |
| 8 | 169.02336 | 453.222384 |
| 9 | 173.7011216 | 467.027056 |
| 10 | 176.998416 | 481.7672803 |
| 11 | 178.821688 | 497.1669635 |
| 12 | 179.1049916 | 512.9313011 |
| 13 | 177.8118615 | 528.7522994 |
| 14 | 174.9366316 | 544.3146717 |
| 15 | 170.5051644 | 559.301998 |
| 16 | 164.5749646 | 573.4030309 |
| 17 | 157.2346615 | 586.3180238 |
| 18 | 148.6028591 | 597.7649561 |
| 19 | 138.8263635 | 607.485528 |
| 20 | 128.0778107 | 615.2508007 |
| 21 | 116.5527306 | 620.8663628 |
| 22 | 104.4660943 | 624.1769089 |
| 23 | 92.04840346 | 625.0701278 |
| 24 | 79.54139068 | 623.4798085 |
| 25 | 67.19340983 | 619.3880866 |
| 26 | 55.25460117 | 612.8267686 |
| 27 | 43.97192431 | 603.8776888 |
| 28 | 33.58415542 | 592.6720737 |
| 29 | 24.31694806 | 579.3889048 |
| 30 | 16.37805758 | 564.2522944 |
| 31 | 9.952828142 | 547.5279059 |
| 32 | 5.200037553 | 529.5184715 |